

Exercise 79

Assume that f is an even function, g is an odd function, and both f and g are defined on the entire real line $(-\infty, \infty)$. Which of the following (where defined) are even? odd?

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|-----------------------|-----------------------|-----------------------|
| a. fg | b. f/g | c. g/f |
| d. $f^2 = ff$ | e. $g^2 = gg$ | f. $f \circ g$ |
| g. $g \circ f$ | h. $f \circ f$ | i. $g \circ g$ |

Solution

Replace x with $-x$ to see which functions are even or odd, assuming f is even and g is odd.

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|---|---|
| a. $f(-x)g(-x) = [f(x)][-g(x)] = -f(x)g(x)$ | $\Rightarrow fg$ is an odd function |
| b. $\frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\frac{f(x)}{g(x)}$ | $\Rightarrow f/g$ is an odd function |
| c. $\frac{g(-x)}{f(-x)} = \frac{-g(x)}{f(x)} = -\frac{g(x)}{f(x)}$ | $\Rightarrow g/f$ is an odd function |
| d. $f(-x)f(-x) = [f(x)][f(x)] = f(x)f(x)$ | $\Rightarrow ff$ is an even function |
| e. $g(-x)g(-x) = [-g(x)][-g(x)] = g(x)g(x)$ | $\Rightarrow gg$ is an even function |
| f. $(f \circ g)(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = f \circ g$ | $\Rightarrow f \circ g$ is an even function |
| g. $(g \circ f)(-x) = g(f(-x)) = g(f(x)) = g \circ f$ | $\Rightarrow g \circ f$ is an even function |
| h. $(f \circ f)(-x) = f(f(-x)) = f(f(x)) = f \circ f$ | $\Rightarrow f \circ f$ is an even function |
| i. $(g \circ g)(-x) = g(g(-x)) = g(-g(x)) = -g(g(x)) = -g \circ g$ | $\Rightarrow g \circ g$ is an odd function |